Statistics

Summer 2023

Lecture 10

Feb 19-8:47 AM

class QZ 11

Use the chart below

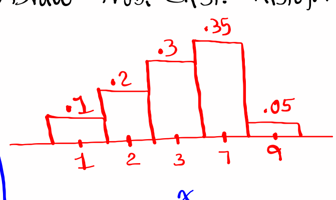
x	$P(x)$
1	.1
3	.2
5	.3
7	.35
9	.05

Total Prob.

1) Find $P(x=9)$
$= 1 - [.1 + .2 + .3 + .35] = \boxed{.05}$ ✓

2) Find $P(x>1)$
$= 1 - P(x=1) = 1 - .1 = \boxed{.9}$ ✓

3) Draw Prob. dist. histogram



$x \to L1$, $P(x) \to L2$

$\boxed{\text{1-VarStats}}$ with L1 & L2

$\mu = \bar{x} = 5.1$

$\sigma = \sigma_x = 2.142$

$n = 1 \leftarrow$ Total Prob.

$\sigma^2 = 4.59 = \frac{459}{100}$

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{\text{4: } \sigma_x}$

$\boxed{x^2}$ $\boxed{\text{MATH}}$ $\boxed{\text{1: ▶Frac}}$ $\boxed{\text{Enter}}$

$\mu \approx 5$, $\sigma \approx 2$

68% Range: $\mu \pm \sigma \to \boxed{3 \text{ to } 7}$

95% Range: $\mu \pm 2\sigma \to \boxed{1 \text{ to } 9} \leftarrow$ Usual Range

Jun 27-11:02 AM

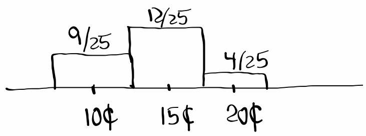
A piggy bank has 2 dimes & 3 nickels.

Take 2 coins with replacement.

NN ND DN DD

10¢ 15¢ 20¢

Total	P(Total)
10 ¢	$\frac{3}{5}\cdot\frac{3}{5}=\frac{9}{25}$
15 ¢	$2\cdot\frac{3}{5}\cdot\frac{2}{5}=\frac{12}{25}$
20 ¢	$\frac{2}{5}\cdot\frac{2}{5}=\frac{4}{25}$



Total → L1, P(Total) → L2

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2 to find

$\mu = 14$

$\sigma = 3.464$

$\sigma^2(\text{exact}) = 12$

$\mu = 14$, $\sigma \approx 3.5$

Usual Range

$\mu \pm 2\sigma = 14 \pm 2(3.5)$

$= 14 \pm 7$

$\Rightarrow \boxed{7 \text{ to } 21}$

Jun 28-7:40 AM

Consider drawing a card from standard deck of playing cards.

You must pay \$5 to play.

If you draw an Ace → I give you \$20.

If you draw a face card → I give you \$10

Any other card, I give you nothing.

Find expected value per bet for the house.

Net gain	P(Net gain)	
5 − 20	4/52	Ace
5 − 10	12/52	face
5 − 0	36/52	other cards

Net gain → L1

P(Net gain) → L2

E.V. = $\mu = \bar{x}$

$\approx \$1.15$

House makes \$1.15 per bet.

Suppose I give you \$50 if you draw an ace.

Net gain	P(Net gain)	
5 − 50	4/52	Ace
5 − 10	12/52	face
5 − 0	36/52	other cards

Now find E.V.

E.V. → −\$1.15

House loses \$1.15 per bet.

Jun 28-7:54 AM

Consider a binomial Prob. dist. with $n=180$ & $p=.75$

1) $q=1-p$
$=1-.75$
$=\boxed{.25}$

2) $\mu=np$
$=180(.75)$
$=\boxed{135}$

3) $\sigma^2=npq$
$=180(.75)(.25)$
$=\boxed{33.75}$

4) $\sigma=\sqrt{\sigma^2}=\sqrt{33.75}$
$=\boxed{5.809}$

Round μ & σ to whole #
$\rightarrow 6$

5) 68% Range
$\mu\pm\sigma=135\pm6$
$\Rightarrow \boxed{129 \text{ to } 141}$

6) 95% Range
$\mu\pm2\sigma=135\pm2(6)$
$\Rightarrow \boxed{123 \text{ to } 147}$

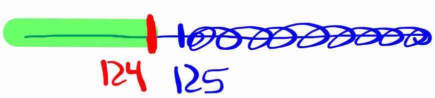
Let x be # of Successes

7) $P(x=140)$
$=\text{binompdf}(180, .75, 140)$
$=\boxed{.049}$

8) $P(x\le141)$
$=\text{binomcdf}(180, .75, 141)$
$=\boxed{.869}$

Jun 28-8:33 AM

9) $P(x\ge125)=1-P(x\le124)=1-\text{binomcdf}(180, .75, 124)$



$=\boxed{.962}$

10) $P(123\le x\le147)$
$=\text{binomcdf}(180, .75, 147)-\text{binomcdf}(180, .75, 122)$
Reduce by 1
$=\boxed{.969}$

Jun 28-8:42 AM

You are making random guesses on a multiple-choice exam with 60 questions.
Each question has 3 choices, but only one correct choice.
Success is to guess the correct ans.

1) $n = 60$ 2) $P = \frac{1}{3}$ 3) $q = \frac{2}{3}$

4) $\mu = np = 60(\frac{1}{3}) = \boxed{20}$

5) $\sigma^2 = npq = 60(\frac{1}{3})(\frac{2}{3}) = \boxed{\frac{40}{3}}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{40/3} = 3.651 \approx \boxed{4}$

7) Usual Range $\mu \pm 2\sigma \Rightarrow$ $\boxed{12 \text{ To } 28}$

8) P(guess exactly 25 correct Ans) $= P(X=25)$
$x = 25$
$= \text{binompdf}(60, 1/3, 25) = \boxed{.042}$

9) P(guess at least 15 correct Ans)
$x \geq 15$
$= P(x \geq 15) = 1 - P(x \leq 14) = 1 - \text{binomcdf}(60, 1/3, 14) = \boxed{.937}$

10) P(guess between 10 and 25 correct Ans, inclusive) =
$P(10 \leq x \leq 25) = \text{binomcdf}(60, 1/3, 25) - \text{binomcdf}(60, 1/3, 9) = \boxed{.931}$
Reduce by 1

Jun 28-8:47 AM

Class QZ 12

Given binomial Prob. dist with $n = 150$, and $P = .6$

Find

1) $P(x = 100)$
$= \text{binompdf}(150, .6, 100)$
$= \boxed{.077}$

2) $P(X < 100) = P(x \leq 99)$
$= \text{binomcdf}(150, .6, 99)$
$= \boxed{.944}$

Jun 28-9:01 AM

Geometric Prob. Dist.: (SG 17)

It is very Similar to binomial prob. dist.

but there is no n.

x where first success happens

$p \to P(\text{Success})$ $q \to P(\text{failure})$

$p + q = 1$, $q = 1 - p$

p & q remain unchanges for any event.

$P(x) = p \cdot q^{x-1}$ $x = 1, 2, 3, 4, \ldots$

$\mu = \frac{1}{p}$, $\sigma^2 = \frac{q}{p^2}$, $\sigma = \sqrt{\sigma^2}$

Jun 28-9:32 AM

Consider a geometric Prob. dist with $p = .5$

$q = 1 - p = \boxed{.5}$

$\mu = \frac{1}{p} = \frac{1}{.5} = \boxed{2}$ $\sigma^2 = \frac{q}{p^2} = \frac{.5}{.5^2} = \boxed{2}$

$P(x) = p \cdot q^{x-1}$

$\sigma = \sqrt{\sigma^2} = \sqrt{2} = \boxed{1.414}$

P(first success happens at 3rd trial)

$= P(x=3) = .5 \cdot (.5)^{3-1} = .5 \cdot (.5)^2 = \boxed{.125}$

Using TI Command $\boxed{2nd}$ $\boxed{VARS}$ ↓ $\boxed{\text{Geometpdf(}}$

$P(x=3) = \text{Geometpdf}(\overset{p}{.5}, \overset{x}{3}) = \boxed{.125}$

P(first success happens before 4th trial)

$x < 4$

$= P(x < 4) = P(x \leq 3)$

$= \text{geometcdf}(.5, 3) = \boxed{.875}$

Jun 28-9:36 AM

You are making random guesses on a multiple-choice exam with 4 choices per question but one correct choice.

$P = \frac{1}{4} = .25$ $\quad q = \frac{3}{4} = .75$

$\mu = \frac{1}{p} = \frac{1}{.25} = \boxed{4}$ $\quad \sigma^2 = \frac{q}{p^2} = \frac{.75}{.25^2} = \boxed{12}$

$\sigma = \sqrt{\sigma^2} \approx \boxed{3.5}$

Usual Range

$\mu \pm 2\sigma = 4 \pm 2(3.5) \Rightarrow \boxed{-3 \text{ to } 11}$

P(first correct guess happens $\underbrace{\text{on 4th}}_{x=4}$ question)

$P(x=4) = \text{geometpdf}(.25, 4) = \boxed{.105}$

P(first correct guess happens $\underbrace{\text{after 4th}}_{x>4}$ question)

$P(x>4) = P(x \geq 5) = 1 - P(x \leq 4)$

$= 1 - \text{geometcdf}(.25, 4)$

$= \boxed{.316}$

P(first correct guess happens $\underbrace{\text{on 2nd}}_{x=2}$ or $\underbrace{\text{5th}}_{x=5}$ question)

$P(x=2 \text{ or } x=5) = \text{geometpdf}(.25, 2) + \text{geometpdf}(.25, 5)$

$= \boxed{.267}$

Jun 28-9:43 AM

A basketball player makes 80% of his FT.

$P = .8$ $\quad q = .2$ $\quad \sigma = \sqrt{\sigma^2}$

$= \sqrt{.3125}$

$\mu = \frac{1}{p} = \frac{1}{.8} = \boxed{1.25}$

$\approx \boxed{.559}$

$\sigma^2 = \frac{q}{p^2} = \frac{.2}{.8^2} = .3125$

P(he/she makes FT $\underbrace{\text{on 2nd}}_{x=2}$ attempt)

$= P(x=2) = \text{geomet pdf}(.8, 2) = \boxed{.16}$

P(he/she makes FT between 3rd and 5th attempt, inclusive)

$P(3 \leq x \leq 5) = \text{geometcdf}(.8, 5) - \text{geometcdf}(.8, 2)$

$= \boxed{.040}$

$P(x=3 \text{ or } x=4 \text{ or } x=5) =$

$\text{geometpdf}(.8, 3) + \text{geometpdf}(.8, 4) + \text{geometpdf}(.8, 5)$

Jun 28-9:54 AM

Poisson Prob. Dist. SG 17

x is # of Successes in an interval

where the average of Successes μ or λ given

Interval

Average # of Successes is given

$x = 0, 1, 2, 3, \ldots$

$$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$$

$e \approx 2.718$

$\sigma^2 = \mu \quad , \quad \sigma = \sqrt{\sigma^2}$

Jun 28-10:35 AM

Consider a Poisson Prob. dist. with $\mu = 4$ in a fixed interval.

$\sigma^2 = 4$

$\sigma = \sqrt{\sigma^2} = 2$

$\Rightarrow$ 68% Range $\mu \pm \sigma$

$= 4 \pm 2 \Rightarrow \boxed{2 \text{ To } 6}$

$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$

$e \approx 2.718$

$$P(x=5) = \frac{4^5}{5!} \cdot (2.718)^{-4}$$

$$= \frac{1024}{120} \cdot (2.718)^{-4} = \boxed{.156}$$

1024 $\boxed{\div}$ 120 $\boxed{\times}$ 2.718 $\boxed{\wedge}$ $\boxed{-}$4 $\boxed{\text{enter}}$

Using TI Command

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ $\boxed{\text{PoissonPdf(}}$

$P(x=5) =$ PoissonPdf(4,5)

$P(x \leq 5) =$ Poissoncdf(4,5) $= \boxed{.785}$

Jun 28-10:39 AM

In average, 150 people attend per movie,

P(exactly 140 attend) $= P(x=140)$

$= \text{Poisson Pdf}(150, 140)$

$= \boxed{.024}$

P(at least 140 attend) $= P(x \geq 140)$

$= 1 - P(x \leq 139)$

$= 1 - \text{poissoncdf}(150, 139)$

$= \boxed{.803}$

Jun 28-10:49 AM

In average, this coffee place has 64 customers between 6:00 AM to 10:00 AM.

Fixed Interval

$\mu = 64$

$\sigma^2 = \mu = 64$ $\quad \sigma = \sqrt{\sigma^2} = \sqrt{64} = \boxed{8}$

68% Range $\rightarrow \mu \pm \sigma = 64 \pm 8 \Rightarrow$ $\boxed{56 \text{ to } 72}$

P(get between 60 and 70, inclusive, customers in that shift)

$P(60 \leq x \leq 70) = \text{poissoncdf}(64, 70) - \text{Poissoncdf}(64, 59)$

$= \boxed{.502}$

P(get 60 or 70 customers in that shift)

$P(x = 60 \text{ or } x = 70) = \text{Poisson Pdf}(64, 60) + \text{Poisson Pdf}(64, 70)$

$= \boxed{.082}$

SG 17 ✓

Exam II $\Rightarrow$ SG 1 – SG 17

Review exam 1

Jun 28-10:53 AM

Data
- 1) Qualitative
  (Non-Numerical)
- 2) Quantitative
  (Numerical)
  - 1) Discrete
    Countable
  - 2) Continuous
    Measureable

Jun 28-11:03 AM

We use Continuous random variable (SG 18-21)

1) Uniform Prob. dist.

2) Standard normal Prob. dist.

3) Normal Prob. dist.

4) Central Limit Theorem

5) Applications

Jun 28-11:05 AM

Class QZ 13

Given a binomial Prob. dist with

$n = 250$ & $P = .8$

1) $q = 1 - P = \boxed{.2}$

2) $\mu = np = \boxed{200}$

3) $\sigma^2 = npq = \boxed{40}$

4) σ (Round to whole #)

$= \sqrt{\sigma^2} = \sqrt{40} \approx \boxed{6}$

5) Usual Range

$\mu \pm 2\sigma = 200 \pm 2(6)$

$\Rightarrow \boxed{188 \text{ to } 212}$

6) $P(195 \leq x \leq 210)$

$= \text{binomcdf}(250, .8, 210) - \text{binomcdf}(250, .8, 194) = \boxed{.763}$

Jun 28-11:14 AM